STABLE SEARCH SPACES FOR A CLASS OF LEARNING CONTROLLERS

KARL MATHIA
Renaissance Sciences Corporation
1351 North Alma School Road, Suite 265
Chandler, Arizona 85224
kmathia@rscusa.com

ABSTRACT
'Learning control' is a term attributed to a broad class of self-tuning processes, where the performance of the controlled system, with respect to a particular task, is self-improved based on the performance from previous similar tasks. However, the critical aspect is the stability of the controlled system during online learning. This also applies to neurocontrol, where neural network learning techniques are used for controller adjustment. Here we present a nonlinear control-theoretic platform that provides stability for a class of learning neurocontrollers by establishing the maximum, stable search space for a given controller parameterization. The theory allows the adjustment of controller parameters and/or structure during the learning process.

INTRODUCTION

We present a nonlinear control-theoretic platform that establishes the maximum, stable search space for a given plant and controller parameterization, and propose it as a basis for designing stabilizing learning controllers for a class of nonlinear plants. 'Learning control' is a term attributed to a broad class of self-tuning processes, where the performance of the controlled system, with respect to a particular task, is self-improved based on the performance for previous similar tasks. The underlying mathematical control theory of the proposed concept is a nonlinear extension of the well-known Youla parameterization of all stabilizing controllers for linear systems [4]. Here we review the theory, outline its appeal for learning control, and apply it to the design of learning controllers with neural networks, often called *neurocontrollers*.

The critical aspect of online learning control and neurocontrol is the stability of the controlled system during the learning process. Several solutions to this stability problem have been proposed. See, for example, [2][3][7]. A related topic is adaptive control. See, for example, [1][6]. But although these methods offer some solutions, they do not parameterize *all* stabilizing controllers for a given plant. Therefore the 'best' controllers for a given plant may not available as a solution. The Youla paremeteraization offers that capability. The linear case, i.e. the Youla parameterization for linear plants [8][10], was proposed for neurocontrol in [9]. That work involved a hybrid frequency domain/neurocontrol algorithm that used offline neural network learning to later adjust a linear frequency domain controller.

The present paper extends the linear approach to nonlinear, online learning control, using the theory published in [4], a nonlinear generalization of [10]. Our control is achieved by defining the parameterized set of all stabilizing nonlinear controllers as the *stable search space* that, for a given plant, guarantees stable online controller learning. The concept is illustrated in Figure 1. All controllers in the known search space S are stabilizing, but do not necessarily produce satisfactory system performance. This is achieved during learning. The initial controller M_{ini} converges to the final, satisfactory controller M_{fin} along the learning trajectory $L(n)$ for every learning iteration n. $L(n)$ cannot leave S. Moreover, for control problems where a single control policy is insufficent, a continuum of policies can be provided to achieve the control objective. For example, when plant and/or environment vary, a new learning process may be initiated. Substantial variations may require adjusting the controller structure for the desired system performance, which is possible within our framework. A variable structure learning algorithm for neural networks is presented, for example, in [5].

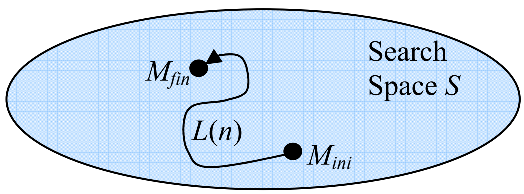


Figure 1. Illustration of a stable search space S and initial and final learning controllers M_{ini} and M_{fin}.

SEARCHING THE SET OF ALL STABILIZING CONTROLLERS

The nonlinear plant model to be controlled is the input-affine system G_{af} defined by

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{h}(\boldsymbol{x}) \end{aligned} \tag{1}$$

$\boldsymbol{f}$, $\boldsymbol{g}$, and $\boldsymbol{h}$ are vector-valued nonlinear functions, $\boldsymbol{x} \in \Re^n$ is the state vector, $\boldsymbol{u} \in \Re^p$ the control input vector, and $\boldsymbol{y} \in \Re^q$ the output vector. We assume $\boldsymbol{f}(\boldsymbol{0}) = \boldsymbol{h}(\boldsymbol{0}) = \boldsymbol{0}$, so $\boldsymbol{0}$ is an equilibrium with $\boldsymbol{u} = \boldsymbol{0}$. The controller parameterization includes two components: an observer-based, fixed stabilizing controller (the 'stabilizer'), and a variable controller component (the Youla parameter) [4].

Stability Analysis. The plant model G_{af} in Equation (1) is stable if $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ is stable. $\boldsymbol{f}(\boldsymbol{x})$ is stable about 0 if there exist a positive definite Lyapunov function $V : \Re^n \to \Re^+$ such that

$$\frac{\partial V}{\partial \boldsymbol{x}}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}) < 0 \quad (2)$$

for all $\boldsymbol{x}$ except 0.

Controllability ('Stabilizability') Analysis. The plant $\boldsymbol{G}_{\text{af}}$ is locally stabilizable if there is a control $\boldsymbol{u} = \boldsymbol{F}(\boldsymbol{x})$ such that $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{F}(\boldsymbol{x})$ is stable. We need to find the vector-valued (or matrix-valued) function $\boldsymbol{F}(\boldsymbol{x})$:

$$\boldsymbol{F}(\boldsymbol{x}) = -\boldsymbol{g}^T(\boldsymbol{x})\frac{\partial V^T}{\partial \boldsymbol{x}}(\boldsymbol{x}) \quad . \quad (3)$$

We consider three solution methods for finding $\boldsymbol{F}(\boldsymbol{x})$: 1) experience and intiution, 2) analytical (closed form) solutions, and 3) numerically solving the Hamilton-Jacobi Inequality (HJI) for the derivative term in Equation (3):

$$\frac{\partial V}{\partial \boldsymbol{x}}\boldsymbol{f} - \frac{\partial V}{\partial \boldsymbol{x}}\boldsymbol{g}\boldsymbol{g}^T\frac{\partial V^T}{\partial \boldsymbol{x}} \le -\gamma(\|\boldsymbol{x}\|) \quad . \quad (4)$$

The function γ is two times continuously differentiable.

Observability ('Detectability') Analysis. The plant $\boldsymbol{G}_{\text{af}}$ is locally detectable if there is a vector or matrix-valued function $\boldsymbol{L}(\boldsymbol{x})$ such that $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{L}(\boldsymbol{x})\boldsymbol{h}(\boldsymbol{x})$ is stable. The closed form solution for $\boldsymbol{L}(\boldsymbol{x})$ can be obtained analytically by solving

$$\frac{\partial U(\boldsymbol{x})}{\partial \boldsymbol{x}}\boldsymbol{L}(\boldsymbol{x}) = -\boldsymbol{h}^T(\boldsymbol{x}) \quad . \quad (5)$$

U is a positive definite Lyapunoc function. Alternatively, the derivative term can be obtained by numerically solving the HJI

$$\frac{\partial U}{\partial \boldsymbol{x}}\boldsymbol{f} - \boldsymbol{h}\boldsymbol{h}^T \le -\sigma(\|\boldsymbol{x}\|) \quad (6)$$

where the function σ is continuous and strictly increasing.

Controller Parameterization. The term 'parameterization' refers to the design of a controller structure with unspecified parameters, as opposed to the design of a specific controller. The parameterization of our observer-based controller includes two parts: stabilizing the unstable plants G_{af} with an observer-based controller, and achieving the desired plant performance with a variable Youla parameter.

a) Designing a fixed, stabilizing controller component: the observer-based controller includes stabilizing state feedback matrix $\boldsymbol{F}(\tilde{x})$ and the output injection function $\boldsymbol{L}(\tilde{x})$,using the state estimates $\tilde{x}$:

$$\dot{\tilde{\boldsymbol{x}}} = f(\tilde{\boldsymbol{x}}) + g(\tilde{\boldsymbol{x}})F(\tilde{\boldsymbol{x}}) - L(\tilde{\boldsymbol{x}})h(\tilde{\boldsymbol{x}}) + L(\tilde{\boldsymbol{x}})y \quad (7)$$

See Figure 2. Note the convention $y_e = y - \tilde{y}$.

b) Designing a variable controller component: the Youla parameter $\boldsymbol{Q}$ in Figure 2 is variable and therefore offers the flexibility needed for learning control. The final controller has the dynamics

$$\begin{aligned}
\dot{\tilde{\boldsymbol{x}}} &= \boldsymbol{f}(\tilde{\boldsymbol{x}}) + \boldsymbol{g}(\tilde{\boldsymbol{x}})\boldsymbol{u} + \boldsymbol{L}(\tilde{\boldsymbol{x}})\boldsymbol{y}_e \\
\boldsymbol{u} &= \boldsymbol{F}(\tilde{\boldsymbol{x}}) + \boldsymbol{Q}(\boldsymbol{y}_e) \\
\boldsymbol{y}_e &= -\boldsymbol{h}(\tilde{\boldsymbol{x}}) + \boldsymbol{y}
\end{aligned} \tag{8}$$

The controlled system is stable if $\boldsymbol{Q}$ is stable [10][8][4]. As such, $\boldsymbol{Q}$ could be a constant gain, a stable linear or nonlinear system, or, as proposed, a neural network. Figure 2 shows the learning controller *M* with fixed and variable component.

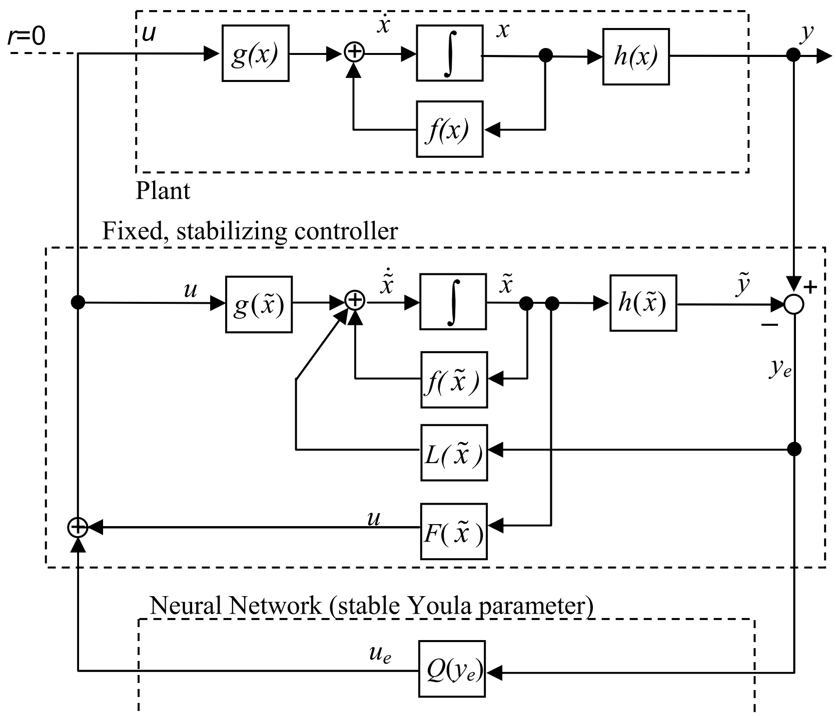


Figure 2. Stabilizing, learning neurocontroller: architecture and flowchart.

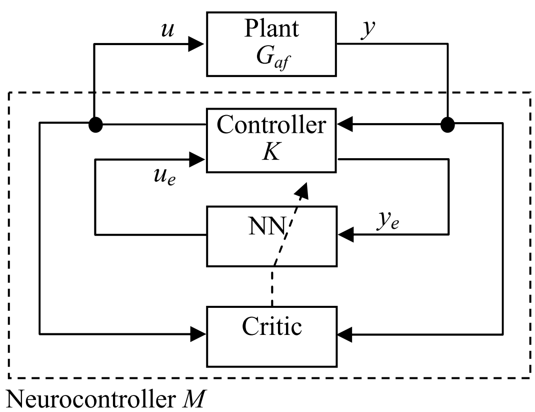
LEARNING NEUROCONTROLLER

The above theory is now used for establishing a control-theoretic platform for a class stabilizing neurocontrollers. The final neurocontroller has two subsystems, i.e. the ‘stabilizer’ *K*, which is fixed in both parameters and structure, and the controller component *NN* that utilzes neural network learning techniques and equips the neurocontroller with learning capabilities. See Figure 2a and Figure 2b. The dynamics of the resulting stabilizing neurocontroller are

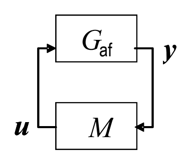
$$\dot{\tilde{\boldsymbol{x}}} = f(\tilde{\boldsymbol{x}}) + g(\tilde{\boldsymbol{x}})F(\tilde{\boldsymbol{x}}) - L(\tilde{\boldsymbol{x}})h(\tilde{\boldsymbol{x}}) + L(\tilde{\boldsymbol{x}})\boldsymbol{y} + g(\tilde{\boldsymbol{x}})NN(\boldsymbol{y}_e), \qquad (9)$$

where a stable neural network *NN* implements the Youla parameter $\boldsymbol{Q}$. Note that the learning dynamics are not included in Equation (9). The neurocontroller *M* utilizes the set of all stabilizing controllers for G_{af} as a search space for improving its performance by adjusting the neural networks's parameters and/or structure, while maintining stability. Note that the set of *all* stabilizing controllers implies the maximum possible search space for a given G_{af}.

Figure 3 offers a *direct* approach for integrating neural network learning techniques with the learning control architecture in Figure 2. The neural network *NN* is an integral part of controller *M*, replacing $\boldsymbol{Q}$. A supervising critic evaluates the control system's performance and adjusts the parameters and/or structure of *NN* accordingly. In an *indirect* approach a neural network adjusts the parameters and/or structure of the parameterized $\boldsymbol{Q}$. Several suitable architectures have been proposed in the litarature, but are are not discussed here, as our intent is to emphasize the availability of a mathematical tool that offers stable online learning for a broad class of nonlinear plants and neurocontrollers.



a)



b)

Figure 3. a) Control system with neurocontroller, and b) simplified diagram.

EXAMPLE

We present an example with scalar system and control variables. Let the plant be defined by

$$\dot{x} = f(x) + g(x)u = x^2 + xu$$
$$y = h(x) = x$$

We desire to find feedback functions *F*(*x*) and *L*(*x*) directly, without creating and then numerically differentiating Lyapunov functions. For example, $F(\tilde{x}) = -\tilde{x} + \tilde{x}^2$ globally stabilizes the plant, while $L(\tilde{x}) = -1 + \tilde{x}$ provides a stable observer. With

$$u = F(\tilde{x}) + NN(y_e), y_e = y - \tilde{y} = x - \tilde{x}$$

we get

$$\begin{aligned}\dot{\tilde{x}} &= f(\tilde{x}) + g(\tilde{x})F(\tilde{x}) - L(\tilde{x})h(\tilde{x}) + L(\tilde{x})y + g(\tilde{x})NN(y_e) \\ &= \tilde{x}^2 + \tilde{x}(-\tilde{x} - \tilde{x}^2) + (-1 - \tilde{x})y_e + \tilde{x}NN(y_e) \\ &= -\tilde{x}^3 + (-1 - \tilde{x})(x - \tilde{x}) + \tilde{x}NN(y_e)\end{aligned}$$

The system is stable if *Q*, i.e. the neural network *NN*, is stable. This can be achieved, for example, with a stable feedforward neural network architecture.

CONCLUSIONS

We presented a nonlinear control-theoretic platform as a basis for a class of stabilizing learning neurocontrollers by defining the stable search space for a given controller parameterization, i.e. the set of all stabilizing controllers for a known plant. The theory allows the adjustment of controller parameters and/or structure during the learning process until a sufficient control system performance is accomplished. An illustrative example was presented.

REFERENCES

[1] Karl J. Astrom and Bjorn Wittenmark, "Adaptive Control," Addison-Wesley, Boston, Massachusetts, 1994.
[2] C. Cox, J. Edwards, R. Pap, and K. Mathia, "Adaptive Semi-Autonomous Robotic Neurocontroller," Proceedings SPIE 1994, pp. 440-449, Orlando, Florida, April 1994.
[3] C. Cox, K. Mathia, J. Edwards, and R. Akita, "Modern Adaptive Control With Neural Networks," Int. Conf. on Neural Network Info. Proc. (ICONIP'96), Hong Kong, Sept. 1996.
[4] W.M. Lu, “A State-Space Approach to the Parameterization of Stabilizing Controllers for Nonlinear Systems,” IEEE Trans. on Automatic Control, Vol. 40, No. 9, pp. 1576-1588, 1995.
[5] K. Mathia, "A Variable Structure Learning Algorithm for Multilayer Perceptrons," in Proc. ANNIE 2005, Intelligent Engineering Systems Through Artificial Neural Networks, C.H. Dagli et al. (Eds.), Vol. 14, pp. 885-891, ASME Press, New York, New York, 2004.
[6] K.S. Narendra and A.M. Annaswamy. *Stable Adaptive Systems*. Prentice-Hall, Englewood Cliffs, New Jersey, 1989.
[7] K.S. Narendra and K. Parthasaraty, “Identification and Control of Dynamical Systems using Neural Networks,” IEEE Transactions on Neural Networks, March 1990.
[8] R. Saeks, J. Murray, O. Chua, C. Karmokolias, and A. Iyer, “Feedback System Design: The Single Variate Case – Part 1,” Circuits, Systems, and Signal Processing, Vol. 1, No. 3, 1982.
[9] R. Saeks, J. Kaiser, and C. Cox, “Neurocontrol and the Youla Parameterization,” Proc. Int. Conf. Systems, Man and Cyb. Syst., Vol. 4, pp. 115-118, Le Touquet, France, 17-20 Oct. 1993.
[10] D.C. Youla, J.J. Bongiorno, H.A. Jabr, “Modern Wiener-Hopf Design of Optimal Controllers – Part II,” IEEE Trans. Automatic Control, Vol. AC-21, pp. 319-338, 1976.