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Cooperative Control of Unmanned Vehicle Formations

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Abstract

We review the enabling theory for the decentralized and cooperative control of formations of unmanned, autonomous vehicles. The decentralized and cooperative formation control approach combines recent results from dynamical system theory, control theory, and algebraic graph theory. The stability of vehicle formations is discussed, and the applicability of the technology concept to a variety of applications is demonstrated.

1 Introduction

A formation of autonomous vehicles refers to a set of spatially distributed vehicles whose dynamic states are coupled through a common control law. The approach is intended for unmanned aerial vehicles (UAVs), but also applies to spacecraft as well as land and water-based robot vehicles. The significance of the presented concept is the possibility of combining several key formation capabilities, including

- formation stability,
- robustness against intermittent communication between UAVs within the formation,
- formation acquisition and regulation,
- formation maneuvers and trajectory tracking, and
- collision avoidance.

Formation control technology for multiple UAVs is desired where it enables capabilities not possible with individual UAVs. The system-level performance of distributed UAVs is particularly important in terms of estimating and controlling the relative UAV positions and orientations [1]-[5][8][10][21]. The presented decentralized and cooperative formation control approach combines recent results from dynamical system theory, control theory, and algebraic graph theory [20][6][7]. The approach offers flexibility and ease of use through its simple definition and specification of a formation, and its communication graph, which describes the communication between UAVs within a formation. For example, a change in the formation's pattern is achieved by simply specifying a new formation configuration vector and presenting it to the formation control law. The control method is applicable to formations of various sizes, to formations with and without leader following, and, in theory, to fleets of spacecraft, or 'formations of formations' [22][23]. Size does not affect formation stability [24][26].

2 Theoretical Framework and Examples

The stable and cooperative formation control method for UAV formations expands previous research results for tight formations. Refer, for example, to [5][6][7][20][21][23][24]. While the feedback control laws previously proposed utilize error measurements between the vehicles' desired and actual locations relative to their neighbors [4][9], the method proposed here defines the control problem as a *stabilizability problem in the class of hybrid control systems*. It is natural to formulate it as a hybrid control problem, as it contains two components:

- the state of the vehicle systems, and
- the potentially discrete interaction between the different vehicles due to intermittent communication.

This section reviews the theoretical framework. Examples and simulations are presented in the following Section 3.

2.1 Vehicle Model, Formation, Formation Stability

The control-theoretical framework commands a UAV formation as a single system. We refer to this concept as 'autonomous' formation control. Vehicles are modeled as linear time-invariant (LTI) systems.

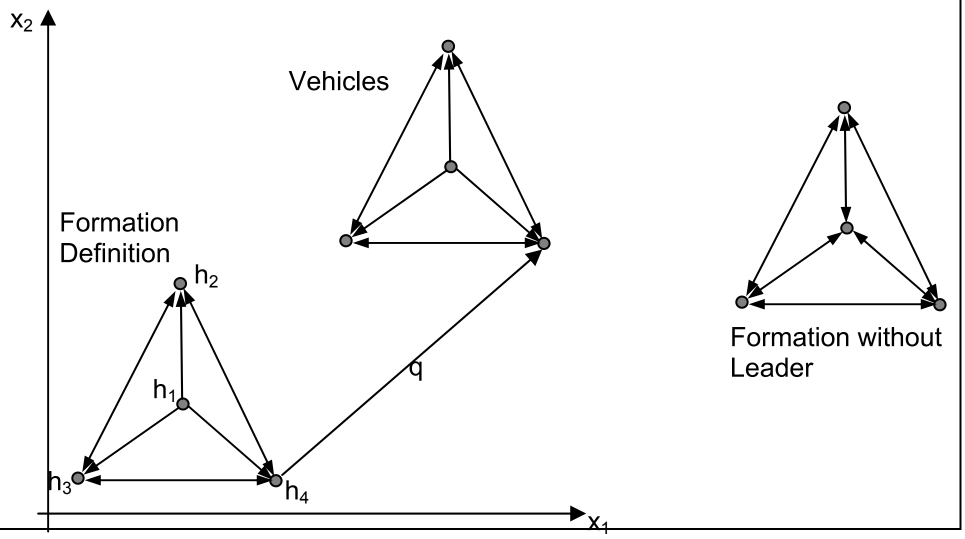
Formation configuration vector: A formation of N vehicles is given by its formation configuration vector

$$h = h_p \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in \Re^{2nN},$$

where '$\otimes$' denotes the Kronecker product. The vector h offers an easy-to-use method to specify formations and is key to our control method. The N vehicles are in formation if there are R^n-valued functions $q(.)$ and $w(.)$ such that $(x_p)_i(k) - (h_p)_i = q(k)$ and $(x_v)_i(k) = w(k)$, i=1,2,...,N. The subscript p refers to the position components of x_i , the subscript v refers to the corresponding velocities [7]. See Figure 1, where the vehicle formation has a leader at its center, with only outgoing communication. The Figure also shows the same formation without a leader, that is an undirected communication graph.

Communication Graph: The communication between vehicles is captured by a directed interaction graph, G =(V,E), where vertex i represents vehicle I and the edge (i, j) denotes the fact that vehicle j receives information from vehicle i. As before, we call i the neighbor of vehicle j. This relationship does not necessarily imply a certain proximity relation between vehicles i and j. In general, the communication graph is not assumed to depend on the distance between vehicles, unless specified. We denote the set of neighbors of a vehicle i by J_i. See Figure 1 for an example interaction graph.

Figure 1 Example: Formation with 4 vehicles and offset vector.



Autonomous Vehicle Formations Problem: given N homogeneous vehicles with linear time-invariant (LTI) dynamics modeled by

$$\dot{x}_i = A_{veh} x_i + B_{veh} u_i \quad ,$$

where $x_i \in \Re^{2n}$ is the state of the *i*-th vehicle that consists of the *n* configuration variables for vehicle *i* and their derivatives, A_{veh} is a 2*n* x 2*n* matrix, B_{veh} is a 2*n* x *n* matrix, and u_i is the control input for vehicle *i*. Grouping all *N* vehicles together in one formation models the formation dynamics as

$$\dot{x} = Ax + Bu \ .$$

Here x denotes the augmented vector of all vehicle states, *A* and *B* are the appropriate size block diagonal matrices, and *u* is the entire system input. The vehicles are assumed to be able to receive/transmit information from/to a subset of the other vehicles. The problem is: find a decentralized control *u* so that all vehicles converge to a pre-specified vehicle formation *h*.

Control u: An admissible control *u* for the vehicle system is called decentralized if it consists of components u_i which depend only on the local relative state information that vehicle *i* receives from its set of neighbors J_i.

Global Reference Frame: All vehicles must agree on a global reference frame. For many applications the Global Positioning System (GPS) can be utilized, for example for satellite formations (‘formation flyers’) in earth-orbits.

2.2 Formation Acquisition and Regulation

Formation acquisition is the asymptotic convergence to a commanded formation configuration (pattern) from some initial condition, i.e. an initial vehicle distribution. The presented autonomous formation control method utilizes a decentralized and cooperative control strategy for formation acquisition, using a class of local feedback control laws without centralized coordination. The objective is to attain and maintain loose and tight formations from arbitrary, but fixed initial conditions. The first control concept was reported in [6][7][24], assuming formations of identical vehicles with linear time-invariant (LTI) dynamics. We also assume directed communication graphs in order to account for limitations due to antenna placement and vehicle orientation. More detailed results are available for undirected graphs.

We use linear formation models controlled by decentralized control laws to achieve stable formation flying [6][7]. These models are based on the linearized dynamics of identical, autonomous vehicles and allow for formations with or without leaders. The formation model with feedback control given by

$$\dot{x} = Ax + BFL(x-h), x(0) = x_0$$

The control law compensates for the formation error *x-h*, with *h* constant for the formation acquisition case. Convergence and stability of formations is determined by both the Laplacian matrix L of the communication graph and the State feedback gain matrix *F*. *L* is defined by $L = D_G - Q$, where Q is the adjacency matrix, and D_G the in-degree matrix of the communication graph. For example, with bi-directional communication between vehicles the rate of convergence to formation is determined by the first non-zero eigenvalue of L [6]. With better communication (more and undirected links) both convergence and stability improve. Figure 2 and Figure 3 illustrate a formation acquisitions using our control method. In the top figure all UAVs, including the ‘virtual leader‘ at the formation center, converge along a near-optimal path to their formation position. In the example the UAVs are modeled by a double integrator.

Figure 2 Formation acquisition for fully connected, bi-directional communication.

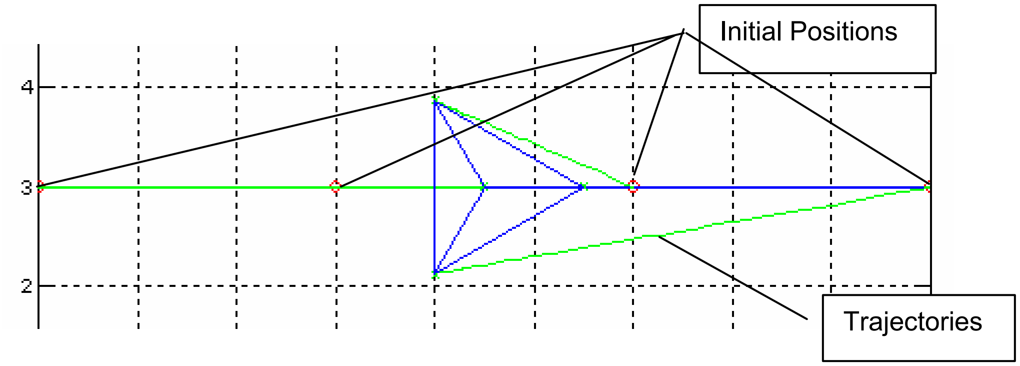
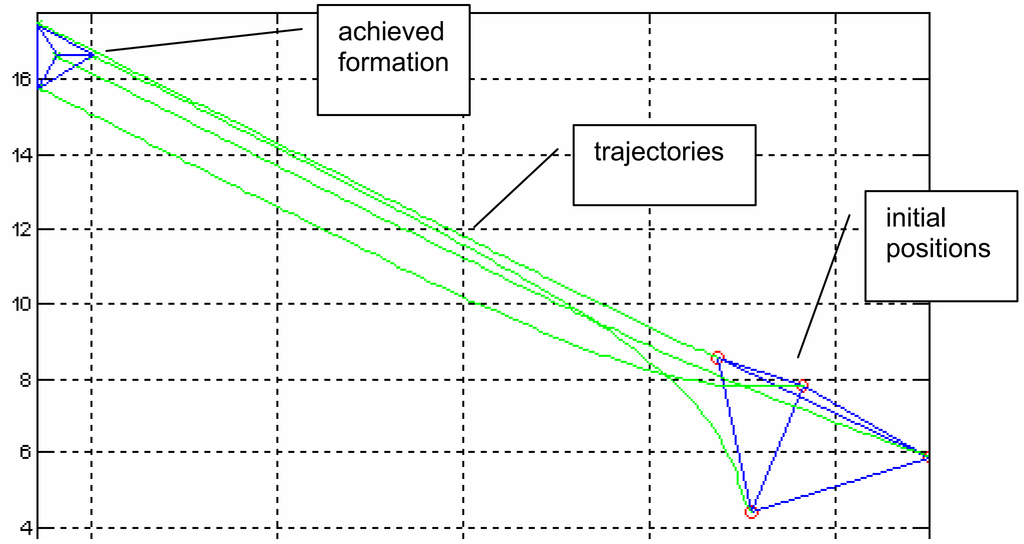


Figure 2 shows a formation acquisition from some initial position as well as the trajectories to the final formation configuration. We also considered scenarios with 'tight' and 'loose' formations, and the varying precision requirements depending on the UAV application. The control tolerance for relative vehicle positions can be varied, and we plan to test this technique for achieving formations of varying 'rigidity'.

Figure 3 Formation acquisition from random initial conditions with a fully connected, bi-directional communication graph.



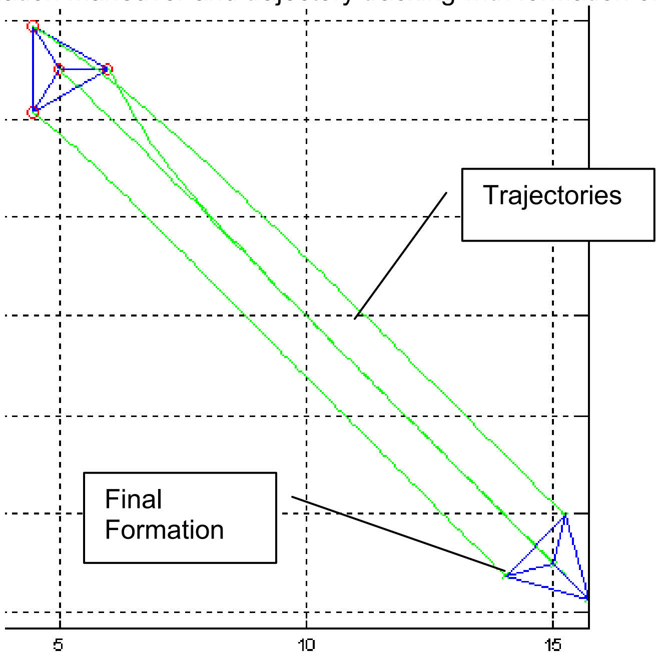
2.3 Formation Maneuvers and Trajectory Tracking

Formation maneuvers will require to track commanded trajectories. The asymptotic tracking problem for linear systems is commonly referred to as finding a control law *u* such that, for a given initial condition, the tracking error goes to zero while the system state *x* remains bounded. To this end we have implemented and simulated a simple tracking controller:

$$\dot{x} = Ax + BFL(x-h) + K_{VL}(y_d - Cy), x(0) = x_0,$$

where y_d denotes the desired trajectory and K_{VL} a controller with appropriate dimensions for the virtual leader only. We introduced the 'virtual leader' for representing a formation reference point. The idea is that only the formation leader tracks the desired trajectory, we have introduced the concept of a ''virtual leader' for improved trajectory tracking, while the formation (the inner control loop) follows and maintains its relative positions as commanded by the vector *h*. Note that the virtual leader is not necessarily an actual spaceship, as the term indicates. The simulation results are shown in Figure 4. The virtual leader tracks a straight trajectory. Note that the formation is gradually orienting itself towards the direction of motion. This is another capability offered by our approach and is accomplished with a third internal 'controller' that rotates the commanded formation configuration *h* successively towards the direction of motion along the commanded trajectory. Trajectory tracking without formation orientation is achieved by simply not rotating *h*.

Figure 4 Formation maneuver and trajectory tracking with formation orientation.



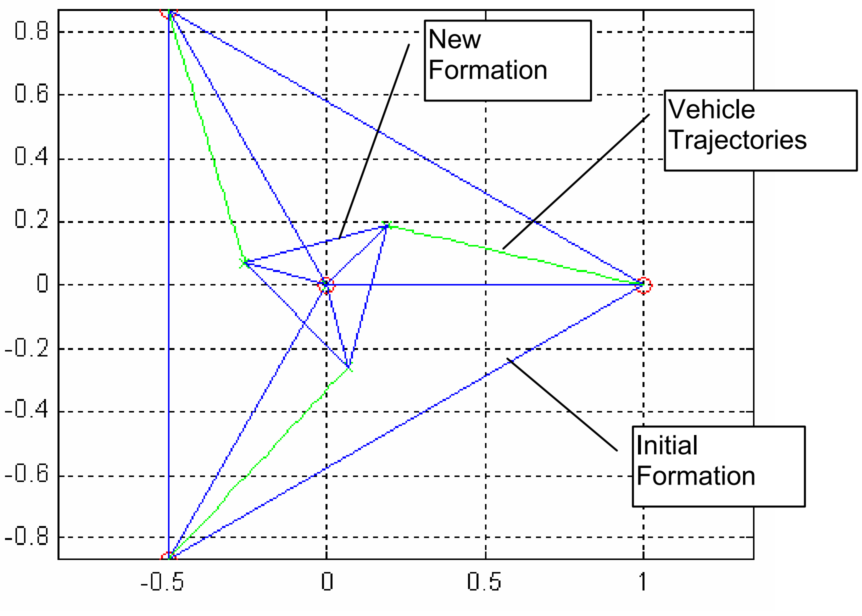
2.4 Formation Reconfiguration

We addressed a control strategy for moving a formation from an initial configuration to a new configuration, for example to perform a new UAV task. The objective is for the UAV to autonomously attain and maintain the new formation in a finite time interval. Reconfiguring a UAV formation is considered a complex formation operation. It requires one formation maneuver or a sequence of maneuvers to obtain a new formation configuration. We have successfully applied our control concept to this problem, as is illustrated by the simulation example in Figure 5. Here the initial formation of three distributed sensors (UAVs or satellites), forming a synthetic aperture antenna, is represented by the vertices of the larger triangular formation. When the antenna requirements change the sensor formation must be adjusted accordingly. In our example it is rotated and reduced in size, converging to the smaller triangular formation. As discussed above, this was accomplished by simply adjusting the formation configuration vector *h*:

$$\dot{x} = Ax + BFL(x - h_{new}), x(0) = x_{old}$$

The control algorithm then converges to the commanded new formation. This example demonstrates the significance of the formation configuration vector *h* as an easily adjustable parameter for formation control.

Figure 5 Formation reconfiguration: rotation as a virtual rigid body with a new synthetic antenna diameter.



2.5 Robustness against Communication Failures

The presented formation control method is stable and robust against intermittent or asynchronous communication between vehicles, i.e. for a dynamically changing communication graph. Such problems arise when the communication between individual UAVs is temporary interrupted, or when UAV leave the relative field of view of other UAV in the configuration.

It was shown in [6][21][23] how vehicles convergence to pre-specified relative positions, i.e. how the formation is accomplished. The control method relies on communication with a certain minimum updated rate between the UAVs within the formation. It is reasonable to assume that the union of communication graphs over some finite time period forms the required connected graph. Then the problem becomes an output stabilizability problem for a hybrid control system (analog dynamics, and discrete communication). The properties of the communication graph are captured by the eigenvalues of the graph's Laplacian matrix. Using graph and control theory the conditions under which such a system can be stabilized can be determined [3]. While many decentralized formation control laws rely on a constant communication graph, where the pattern of open links remain constant throughout a mission, our stability and robustness analysis for the presented control system shows robustness against intermittent and asynchronous communication between team members. A varying communication graph does not affect stability and robustness, as long as it remains connected often enough [4][26]. The standard decentralized model then becomes

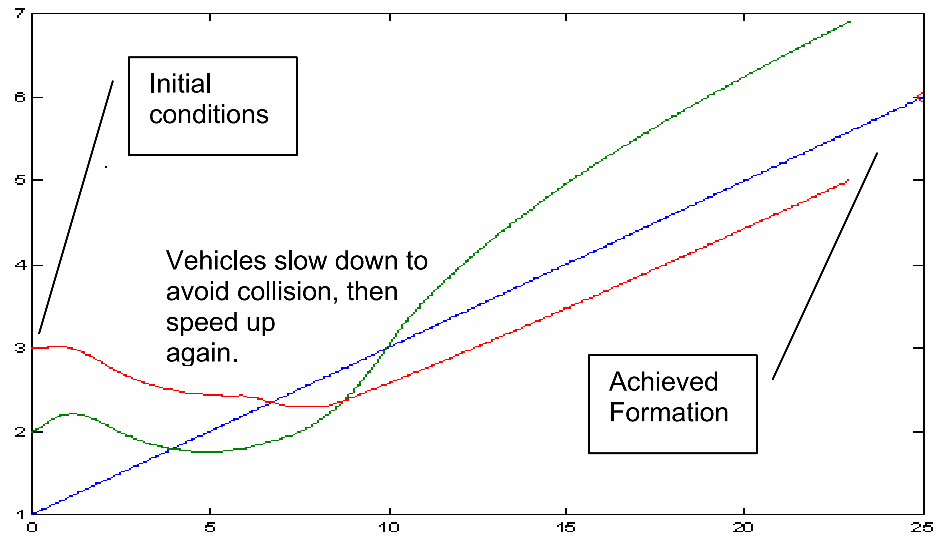
$$\dot{x} = Ax + BFL(k)(x - h)$$

where $L(k)$ denotes the changing graph Laplacian chosen from a finite set of options, and h denotes the commanded formation configuration. The parameter k , i.e. the frequency of graph changes, is a result of the communication conditions. A preliminary stability analysis for such systems is in [23]. Stability can be enhanced by using virtual state updates while communications are down and keeping a virtual team model on each member's computer.

2.6 Collision Avoidance

Collision avoidance is needed e.g. in case of temporary single UAV failure within the formation. We have designed and simulated a simple "potential function" to separate the vehicles at a given threshold. Vehicles of closer proximity avoid each other by slowing down and changing course. This method is utilizes well-known ‘potential field correction’ terms. Thus collision avoidance is achieved by “repellent” forces modeled as a secondary Laplacian function whose goal is to separate vehicles that come within the safety sphere of each other. Our simulation analysis shows promising results. Figure 6 shows the resulting collision-free convergence from an initial configuration.

Figure 6 Collision avoidance of a formation with leader (straight trajectory) using an artificial ‘potential field’.



3 Conclusion

We have reviewed a theory for the decentralized and cooperative control of formations of unmanned, autonomous vehicles. The decentralized and cooperative formation control approach combines recent results from dynamical system theory, control theory, and algebraic graph theory. First we focused on vehicle and formation models, addressed formation stability, and then discussed the suitability of the presented control method for robustness against intermittent communication between UAVs within the formation, formation acquisition and regulation, formation maneuvers and trajectory tracking, and finally collision avoidance. Various simulation examples demonstrated the presented theoretical framework.

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